Math 53, Discussions 116 and 118 **2022-10-10**

Local maxima and minima

Questions

Question 1. Suppose that f is a function such that $\nabla f(2,4) = \langle 0, 0\rangle$ and

$$\begin{bmatrix} f_{xx}(2,4) & f_{xy}(2,4) \\ f_{xy}(2,4) & f_{yy}(2,4) \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -2 \end{bmatrix}.$$

Is $(2,4)$ a local minimum, local maximum, saddle point, or is there not enough information to tell?

Question 2. Let $f(x, y)$ be a (nice) function whose domain is the entirety of $\mathbb{R}^2$. Which of the following statements are true?

(a) f can have a local maximum but no absolute maximum.
(b) f can have an absolute maximum but no local maximum.
(c) f must have an absolute maximum when constrained to $xy = 1$.
(d) If f has a local minimum or maximum at a point, then $\nabla f = \mathbf{0}$ at that point.
(e) If $\nabla f = \mathbf{0}$ at a point, then f has a local minimum or maximum at that point.
(f) f must have an absolute maximum when constrained to $x^2 + y^2 \leq 1$.

Question 3 (Stewart §14.7 #40)**.** Consider the function $f(x, y) = 3xe^y - x^3 - e^{3y}$.

(a) f has exactly one critical point. Find it.
(b) Using the 2nd derivative test, show that this critical point is a local maximum.
(c) Is this point also an absolute maximum?